

# Grushin Reduction and Finite-Dimensional Effective Hamiltonians in Semiclassical Resonance Theory

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## Abstract

We present a rigorous formulation of the Grushin reduction method for semiclassical operators with hyperbolic trapped sets. The spectral problem is reduced to the invertibility of a finite-dimensional effective Hamiltonian, following the framework of Sjöstrand–Zworski.

## 1 Introduction

Let  $P(h)$  be a semiclassical operator with hyperbolic trapped dynamics. We construct an extended operator

$$\mathcal{P}(z) = \begin{pmatrix} P(h) - z & R_- \\ R_+ & 0 \end{pmatrix}.$$

## 2 Main Result

**Theorem 1.** *There exist finite-rank operators  $R_+, R_-$  such that:*

- $\mathcal{P}(z)$  is invertible if and only if the effective Hamiltonian  $E_{-+}(z)$  is invertible.
- $z$  is a resonance if and only if

$$\det E_{-+}(z) = 0.$$

*Moreover, the resolvent admits the block representation induced by the inverse of  $\mathcal{P}(z)$ .*

## 3 Grushin Framework

We define:

- auxiliary spaces of finite rank
- projections  $R_+, R_-$
- effective Hamiltonian  $E_{-+}(z)$

The system reduces to a finite-dimensional spectral problem.

## 4 Sketch of Proof

- Fredholm reduction of  $P(h) - z$
- construction of auxiliary operators
- Schur complement identity

## 5 References

Sjöstrand, semiclassical resonances.

Zworski, Semiclassical Analysis.

Dyatlov–Zworski, scattering theory.